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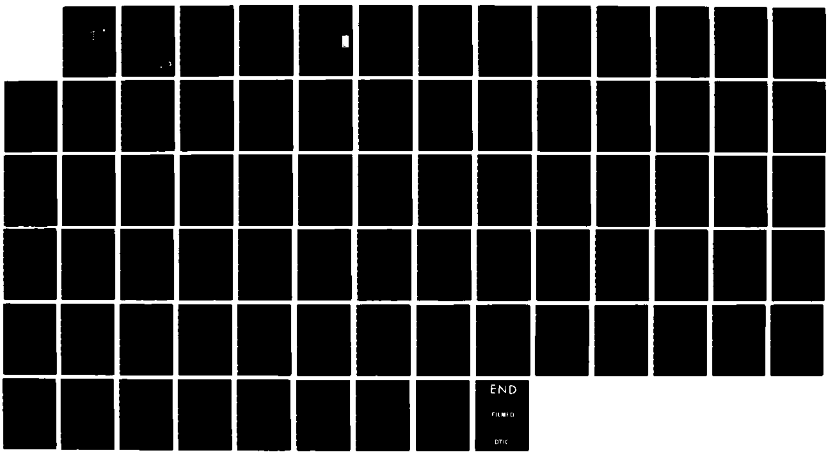

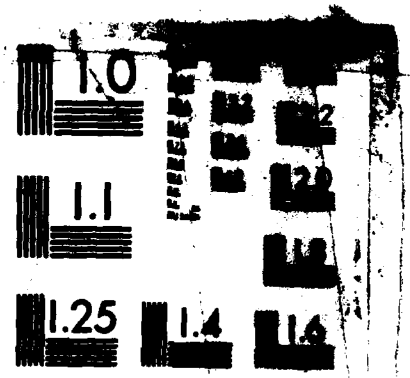

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AD-A158 203

SAMPLING-RATE EFFECTS ON RADAR-DERIVED

RAINFALL ESTIMATES

A Thesis (62 Pages)

by

JEFFREY LYNN FORNEAR , 1Lt, USAF

Submitted to the Graduate College of
Texas A&M University
in partial fulfillment of the requirement for the degree of

MASTER OF SCIENCE

August 1985

Major Subject: Meteorology

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: AFIT/CI/NR 85-67T
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): Sampling-Rate Effects on Radar-Derived Rainfall Estimates
5. TYPE OF REPORT & PERIOD COVERED: THESIS/DISSERTATION
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Jeffrey Lynn Fornear
8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: AFIT STUDENT AT: Texas A&M
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: AFIT/NR WPAFB OH 45433
12. REPORT DATE: August 1985
13. NUMBER OF PAGES: 62
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
15. SECURITY CLASS. (of this report): UNCLASS
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report): APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
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ABSTRACT

Sampling-Rate Effects on Radar-Derived

Rainfall Estimates. (August 1985)

Jeffrey Lynn Fornear, B.S., University of Utah

Chairman of Advisory Committee: Dr. George Huebner

This study investigates the errors due solely to sampling intervals that occur with radar-derived total rainfall estimates. The study was limited to nine cold-front passages over eastern Texas, in the Fall of 1984. Digitized, 10.3 cm wavelength radar observations were recorded using a one minute sampling-rate. Total rainfall estimates, for 10 km by 10 km areas, based on these data were considered "ground truth" totals.

Sample-rates, ranging from 5 to 60 minutes, were applied to the recorded data to calculate total rain estimates for each sample rate. These derived rain totals were compared to the "ground truth" totals, with the differences referred to as "errors." These errors were plotted against the sampling-rate. They ranged from over 100% for sample intervals greater than 50 minutes, to less than 25% for intervals less than 15 minutes. The errors were also plotted against the number of samples taken. There was no significant increase in estimate accuracy when greater than seven samples were taken per 80 minute period.

Other variables, the mean rain rate, total rain, sequential variability, storm width, and storm speed of movement, were found to have very low correlations with the errors. Analyses of variances done on subdivisions of the storm width, storm speed, and mean rain rate

variables proved inconclusive because of small, unbalanced sample sizes. Regression analyses were used to develop the "best" models, using error as the dependent variable. The resulting equations relate the errors to the sampling-rate and the number of samples taken. These models were then used as predictors of the expected errors in total rain estimates. The predictions are applicable to individual, 10 km by 10 km area, total rain measurements.

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TABLE OF CONTENTS

LIST OF TABLES

Table | Page

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Overview

Radar-derived values of total rainfall are, in addition to many other factors, a function of the time interval between radar samples. Most users of such information have little quantitative knowledge regarding the errors in total rainfall estimates that can occur due to variations in the rainfall rate during the intervals between samples. Sampling intervals usually range from 5 to 30 minutes.

In addition to the desirability of determining these errors, there is a specific need in the military for such information. A tactical military weather radar must operate no longer than is absolutely necessary because it presents itself as a target through electromagnetic radiation. Such a radar used for military hydrological purposes needs to accumulate precipitation data sufficient to derive the total rainfall over the area of interest.

A derivation of such errors due to the variations in the sampling interval is of major importance to the military as well as the scientific community.

Objectives

This study was undertaken to identify and quantify the differences

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This study follows the style and format of the Journal of Climate and Applied Meteorology.

between total rain estimates as functions of the radar sample-rates. The total rain estimate derived using radar data recorded at a one-minute sample-rate was assumed to be the "ground truth" estimate. Total rain estimates using other sampling-rates were compared to this "ground truth." The resulting differences are referred to loosely as "errors" in this study. The specific objectives are as follows.

(1) Complement and extend the applicability of previous, similar studies that were based on only rain gage network data.

(2) Use descriptive statistical techniques to describe and place bounds on the expected error associated with specific radar sampling intervals.

(3) Develop regression relationships that can be used to predict the expected error when given a certain sampling interval and field determinable parameters, such as storm depth and speed of movement.

Previous Research

Wilson (1964) showed that the sampling rate used to observe precipitation events can contribute significant errors to the overall rainfall estimates. This is patently clear to anyone but more importantly, just what factors have a bearing on these errors and just how much can be attributed to each such factor?

The results of this study can be applied in several meteorological, hydrological, and agricultural specialties. Models for weather modification verification and streamflow or flood control forecasting use integrated precipitation over areas as important inputs (Larson, 1974; McGuiness, 1963). Outputs from these models can be no more accurate

than the inputs. Therefore, any research that can describe and quantify the expected errors in radar rainfall measurements would be of benefit (Brandes, 1975; Jatila and Puhaka, 1973a,b).

In addition, a measurement of these errors is important to the survivability of tactical weather radars now in use. When such a radar is operating, or active in its hazardous battle environment, the radar beam can act as a homing beacon for enemy rockets or missiles. Thus, the less frequently the radar is active, the better its chances of survival. If the weather officer has knowledge of the relative accuracies of different sampling rates, he can then use the minimum rate necessary. To achieve this, a delineation of the expected statistical bounds of error for certain sampling intervals is needed.

Previous studies, Huff and Neill (1957), and Linsley and Kohler (1951), looked at sampling-rate caused errors with extensive raingage networks. Neill (1953) worked with raingage data from 8 storms. He related the standard error of the estimate to the total storm rainfall and the sampling interval used. His equation is

$$E_s = 4 \times 10^{-3} R_t^{1.13} T^{1.29} \tag{1}$$

where E_s is the standard error of the sampling interval estimate, R_t is the total integrated rainfall in inches, and T is the sampling interval in hours.

Mueller (1957) worked with one-minute data from Neill's 8 storms plus 12 more, of varied synoptic types. He investigated a measure of the rate of change of rain intensity with time, which he called sequential variability, written as

$$D = \frac{|R_1-R_2| + |R_2-R_3| + \ldots + |R_n-R_{n+1}|}{N - 1} \tag{2}$$

where D is the sequential variability in mm/h, R is the mean rainfall rate in mm/h for the minute indicated by the subscript n, and N is the total number of minutes sampled. This quantity will be used in this study.

However, in considering the best multiple correlation coefficient, he concluded that a simple relationship between the standard error of the total rainfall estimate, the total mean storm rainfall, and the sampling interval was the best estimate of sampling error. Thus Mueller's equation is

$$E_s = 1.05 \times 10^{-3} R_t^{.57} T^{1.54} \quad . \tag{3}$$

This equation showed the standard error to be less dependent on total storm mean rainfall than Neill's. Mueller attributed this to the difference in storm types.

Huff (1970) was not concerned with sampling rates but related the mean rainfall rate and the gage density to the sampling error in the equation

$$E = - 1.522 R_m^{.87} G^{.52} \tag{4}$$

where E is the sampling error in inches, R_m is the areal mean rainfall rate in inches per hour, and G is the gage density in square miles per gage. This was done for 29 storm samples over a gage network of 100 mi square. He determined that the mean rainfall rate, or intensity, was an

important variable when assessing errors.

Wilson (1970) also used rain gages to infer expected errors in radar rainfall estimates as functions of sampling interval and size of the integration area. While showing the expected increasing error due to increasing sampling interval length, it became apparent that there was a large effect caused by the size of the total integrated area.

CHAPTER II

PROCEDURE

Data Collection

The WSR/TAM-1, 10.3 cm wavelength radar was used to collect the rainfall data. Digitized radar data were recorded for later playback and analyses.

The physical range of this study, seen in Fig. 1, consisted of a 300 km by 300 km area divided into four quadrants, with the radar in the center. This range of 150 km radius about the radar limited volume filling or beam height errors. This large area was subdivided into a 10 km by 10 km grid, which is the military's basic hydrologic unit.

With the aid of a data processing program a determination was made of the average radar reflectivity factor for each 10 km by 10 km grid area. This average reflectivity value is then converted to an instantaneous average rainfall rate for the grid area with the often used relation

$$R = \left(\frac{Z}{200}\right)^{.625} \tag{5}$$

where Z is the average grid area reflectivity factor in mm^6/m^3 and R is the rainfall rate in mm/h.

The study was limited to cold front type precipitation events for two reasons:

(1) This type of system occurred most frequently in this area during the data collection period from September to November of 1984.

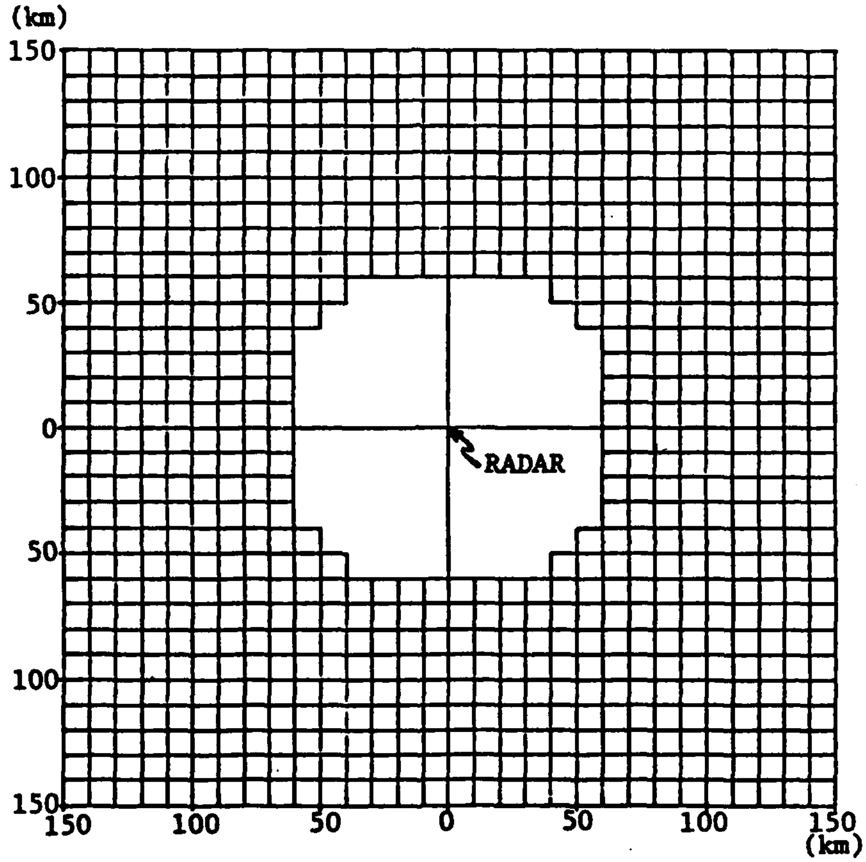


Fig. 1. Illustration of the data grid of 10 km by 10 km areas.

The actual recorded data was from nine different storms.

(2) A somewhat homogeneous type of line shape of radar echo was needed to measure directly several of the physical characteristics of the storms.

The storms were observed by the radar and recorded at an antenna rotation rate of 1 rpm. This allowed information on every grid area every 60 seconds. This rate was then somewhat comparable, time-wise only, to the previously mentioned studies because they used one-minute recording rain gages. The recorded one-minute radar data was then considered "ground truth" or the best possible estimate of rainfall. Total rain estimates based on this one-minute data were also considered the "ground truth" for comparisons with other sample-rate estimates.

The 1 rpm recording rate forced an extrapolation of the rainfall rate data at several of the sample intervals. This extrapolation became necessary because the digitized radar tapes recorded for 85-90 minutes at this antenna rotation rate. With this time span in mind it was decided to process all tapes for a uniform 80 minute time span. The problem then was how to choose the sample intervals to use on the data. Only the intervals of 5, 10, 20, and 40 minutes fit evenly into the 80 minute tape time. While the errors beyond the 40 minute interval were of interest it was desirable to have more data points at the shorter sample intervals. If the 80 minute observation was included as the last data point for all extrapolated intervals then the stated sample intervals would not accurately reflect the actual time intervals with which the data was observed. In that case the stated 50 minute sample interval would actually consist of an end observation interval of 30 minutes. Therefore it was decided to extrapolate the last rain rate measured by a full sample interval to the 80 minute end time. For example, the 50 minute sample was based on a first observation at starting time, a second at the 50 minute point, and then this 50 minute rate was used as

the rain rate at the 80 minute end point. This was thought to be the best way to handle the dilemna because without actually taking a radar observation at the tape end point it would not be known if the rain rate had increased or decreased, both being equally possible. Over a large sample the mean of the data errors due to these extrapolations should be very small because of the equal possibility of under-estimating or over-estimating any given rain rate. The sample intervals of 5, 10, 15, 20, 25, 30, 40, 50, and 60 minutes were used in this study. They are shown with their amounts of extrapolated data in Fig. 2.

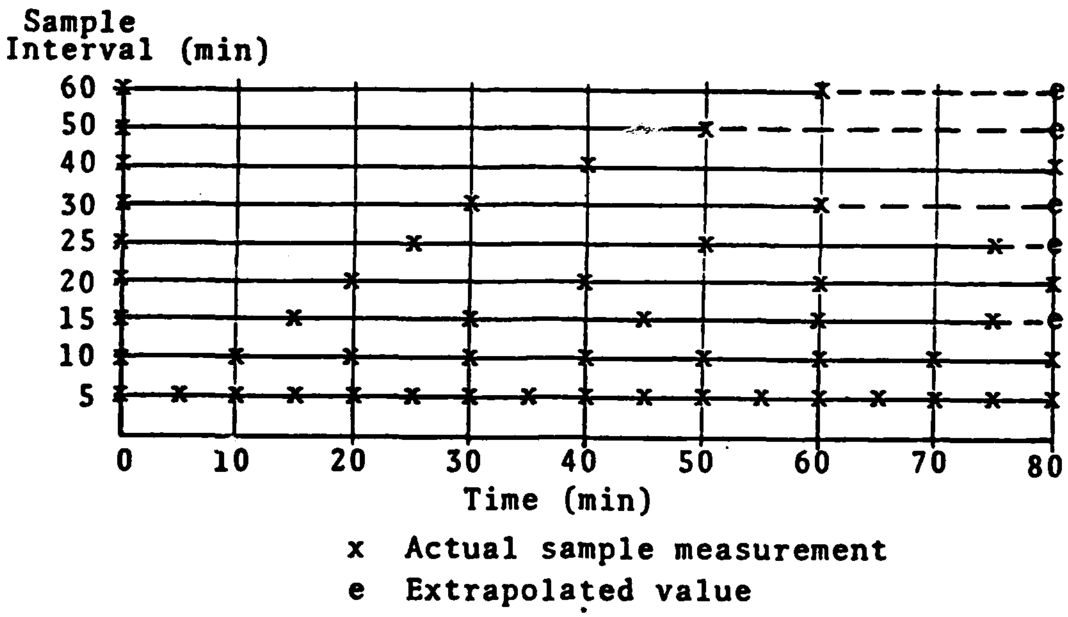


Fig. 2. Illustration of sample interval points of measurement and extrapolation within an 80 min time span.

The selected sample intervals consisted of a set number of samples because of the fixed 80 minute data tape time in this study. These numbers of samples provide a different measure of how the storm was sampled. The 50 and 60 minute sample intervals are different because their measurements are taken at different time points. Although if these two intervals are compared by the number of samples, they are the same, they both allow two actual samples in the fixed time span. The sampling error of a total rain estimate ultimately depends on how well the sampling technique can define the temporal rain profile. In this way the number of samples is important because they obviously have a direct effect on how well that profile is defined. For this reason relating numbers of samples to errors of rain estimates could develop useful predictive type relations. Using the selected sample intervals of 5, 10, 15, 20, 25, 30, 40, 50, and 60 minutes allow 17, 9, 6, 5, 4, 3, 3, 2, and 2 samples respectively for each total rain estimate made in the 80 minute time span.

Variable Selection

There are several variables that could possibly help to explain the observed errors when increasing the sampling interval. The grid instantaneous rainfall rate R in mm/h is the basic unit used to compute these variables. The first three variables were calculated by a program that processed the radar data and gave the variables of interest for each grid area within one quadrant. These variables are as follows:

(1) The total integrated rainfall, R_t (mm), is

$$R_t = \sum_{n=1}^{N-1} \frac{T(R_n+R_{n+1})}{2} \quad , \tag{6}$$

the grid total rain for the entire sampled time.

The mean rainfall rate, R_a (mm/h), is

$$R_a = \frac{\sum_{n=1}^{N} R_n}{N} , \tag{7}$$

the mean grid rain rate over the entire sampled time.

The sequential variability D in mm/h is the rate of change of rain intensity with time. It is shown in Eq. (2).

The following three quantities were determined from tracings of the echoes first and last position:

(1) The horizontal depth of the precipitation area, in km, is measured along the direction of movement. This depth is the arithmetic mean of the distances from the area's leading edge to its training edge, measured at the first and last sample times.

(2) The horizontal depth of the sampled portion of the precipitation area, in km, is the portion of the precipitation area that has passed over the sampling point during the sampling time. This depth is measured as the distance the area's leading edge has moved during the sampled time.

(3) The precipitation area speed, in km/h, is the average rate of movement during the sampling time. The speed is calculated as the arithmetic mean of the distinaces that the leading and training edges moved during the sampled time, divided by the sampled time.

Raw Data Analysis and Products

In analyses of the digitized radar data tapes the previously

mentioned calculations were done for the variables of interest. The calculated values were then plotted for each grid area over the 150 km by 150 km quadrant of interest.

A portion of such a plot for a sample interval of one minute is shown in Fig. 3. Such derived results were plotted for each grid area. The upper number is the grid rain rate in mm/h at the last sample time while the second number is the mean grid rain rate in mm/h for the total sampled time. The third number in each grid area is the grid's total integrated rainfall in mm for the entire sampled time and the lower number in each box is the sequential variability in mm/h. These values, derived using the one-minute sample data, were then considered "ground truth" estimates for each grid area.

A somewhat similar plot was used for each of the sample intervals ranging from 5 to 60 minutes. The upper number is the rain rate in mm/h at the last sample time while the sample interval's total integrated

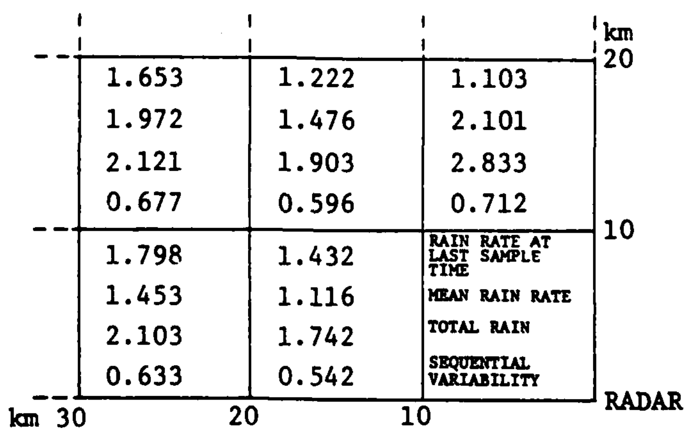


Fig. 3. Illustration of plotted variables within grid areas for the 1 min sample interval.

rain in mm is the lower number. This plot is shown in Fig. 4. Such a plot was generated for each sample interval.

The total integrated rainfall estimate for each sample interval was then compared to the "ground truth" one-min interval total rainfall on a grid by grid basis. From this comparison the error was calculated and expressed as the absolute percent error of the total rain estimate for a given sample interval. This quantity was calculated as

$$APE = \frac{100 \left| R_{t(n)} - R_{t(1)} \right|}{R_{t(1)}} , \qquad (8)$$

where APE is the absolute percent error, R_t is the total integrated rain calculated for the subscripts n and 1 which refer to the number of minutes in the sample interval used.

The recorded radar data tapes were played back to make tracings of the storm echoes. Fig. 5 shows a typical tracing of the echo positions

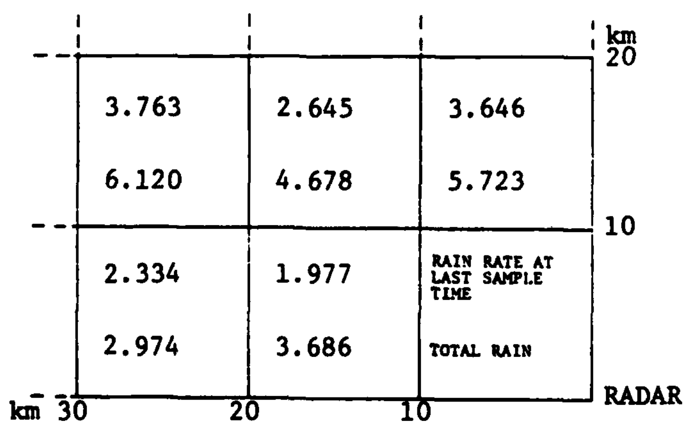
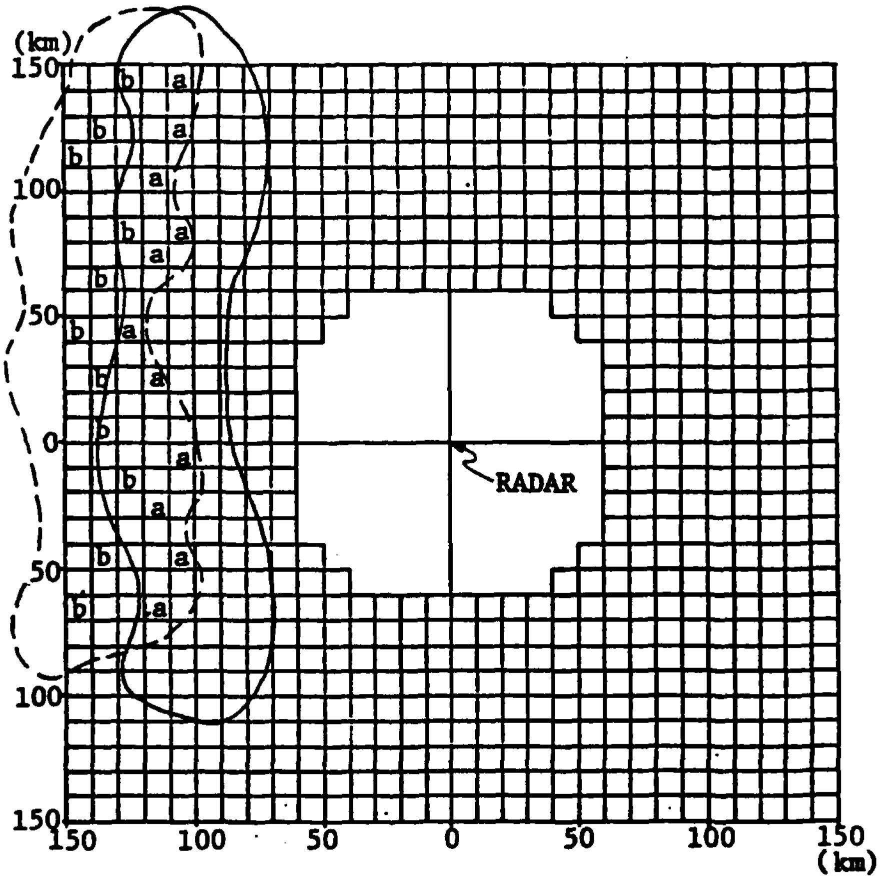


Fig. 4. Illustration of plotted variables within grid areas for the 5 to 60 min sample intervals.



--- Echo position at first data processing time
——— Echo position at last data processing time
a ADATA data set point
b BDATA data set point

Fig. 5. Illustration of overlaid precipitation echoes and typical grid areas selected as data points.

at the first data processing time (dashed line) and the last processing time (solid line). The tracings were constrained to enclose areas of precipitation rates greater than 2.4 nn/h. These tracings grid areas were selected along the storm's leading edge. Two different sample sets of data were selected from each storm. One sample set, ADATA, was selected with the condition that the grid areas were within the precipitation area for the entire sampled time. This set was comprised of values from 103 grid areas. The other set, BDATA, had grid areas under the traced echo at the beginning sample time but not always under the echo at the last sampled time. This data set had 89 grid areas.

Since the mean rain rates were calculated by averaging over the total sampled time, rain rates in BDATA are not valid measurements. The only other difference between the sets was that BDATA grid areas were generally 10 to 20 km further into the storm, away from the leading edge. The two data sets were assumed to be independent samples. Two large data sets were useful for comparisons of mean errors.

Huff (1970) stated that rainfall measurement variables are not normally distributed. This makes statistical analyses difficult because analyses of variance and multiple comparison tests require assumptions of normality and equal variances (Ostle and Mensing, 1982). Huff (1970) found that using log transofrmations of the rainfall variables was the best method of approaching normalization of the data. Natural log transformations of the rainfall errors and rainfall variables were used for all statistical analyses of the data in this study. All statistical testing in this study was done at the 95% significance level.

The total sample in numbers of grid areas was very large by

statistical standards. The data were from 192 grid areas. The errors were determined for nine different sample intervals for each area, or a total of 1728 calculated errors. For a sample this large the results can be generalized fairly accurately with descriptive statistics (Ostle and Mensing, 1982).

CHAPTER III

STATISTICAL ANALYSES

Descriptive Statistics

The absolute percent error described in Eq. (8) was decidedly the most practical measure of the difference between a certain sample interval's total rain estimate and the 1 minute sampled "ground truth" total rain. The percentage part of this type of measurement scaled the errors in an important way that made the errors of a light rainfall comparable to that of a heavy rain. Taking the absolute value of the percentage error was necessary because there is no possible way of knowing if a radar is under-estimating or over-estimating the rainfall at any specified point and time. Thus, in interpreting the results presented here it must be kept in mind that the true errors could be positive or negative.

The arithmetic mean, over all grid areas, of the absolute percent errors for each sampling interval and their associated standard deviations were calculated for each data set. A summary of each individual data set's statistical measures of the errors are shown in Table 1. This table shows that the BDATA set of observations had larger mean errors than the ADATA set, at all sample intervals except 10 min. The BDATA set also had larger standard deviations at all sample intervals except 10 and 20 min.

Are the two data set's mean errors of total rain estimates statistically different? The sets came from different grid areas within nine storms. If it can be shown that they are not significantly different

Table 1. Summary of statistical values of absolute percent error by sample interval (min) for the individual data sets.

Sample interval (min)	Minimum error (%)	Mean error (%)	Maximum error (%)	Standard deviation
		Data set = ADATA		
5	0.00	1.23	5.76	1.16
10	0.00	4.56	26.16	5.38
15	0.00	7.52	38.56	7.17
20	0.07	11.77	131.44	14.53
25	0.31	14.53	71.29	12.72
30	0.53	23.43	138.60	22.05
40	0.66	29.49	104.97	24.78
50	0.24	41.09	128.85	28.77
60	1.00	42.55	106.36	27.71
		Data set = BDATA		
5	0.00	1.70	32.31	3.43
10	0.00	4.31	26.22	4.65
15	0.00	8.61	51.33	9.58
20	0.23	12.87	52.56	11.38
25	0.00	18.15	82.60	18.30
30	0.10	26.66	177.43	28.68
40	0.90	34.06	149.18	30.50
50	0.26	43.03	181.40	39.25
60	0.30	38.80	254.88	40.58

then one plot of combined mean data would be more representative of the recorded data because the sample size would be effectively doubled. An analysis of variance was used to compare the mean errors of the data sets. This statistical test assumes equal variances and a normal distribution, thus it required the log transformation of the errors. The log transformed errors were then tested in two different ways.

First a t-test was done by sample intervals to compare the mean errors of the sets. The ADATA set had 103 observations averaged for

each sample interval, while for the BDATA set each mean was of 89 observations. The results of the tests are shown in Table 2. The overall results of this test show the highest t-statistic to have a level of significance of p = .1676. From this it was concluded that there were no significant differences between the mean errors of the two data sets when they were compared by sampling intervals.

The second t-test was a comparison of each data set's mean error averaged over all sample intervals, shown as the lower line in Table 2. The results were a t = 0.8369 with a level of significance of p = 0.4028. The conclusion was that there was no significant difference in the overall mean errors of the data sets.

Table 2. ADATA and BDATA t-test comparisons of mean absolute percent errors by sample interval and by overall data set means.

Sample interval (min)	t-statistic	p-value	Conclude means are:
5	-1.3852	.1676	EQUAL
10	.2598	.7953	EQUAL
15	- .3597	.7195	EQUAL
20	- .7735	.4402	EQUAL
25	- .5207	.6032	EQUAL
30	- .4955	.6208	EQUAL
40	-1.3630	.1745	EQUAL
50	.9749	.3310	EQUAL
60	- .4762	.6345	EQUAL

From the above t-tests it is seen that the mean errors of total rainfall estimates of the two data sets were not statistically different. This then allows the sets to be combined and a mean error of

the rain estimates calculated for each sample interval. These overall mean errors were plotted in Fig. 6. The actual statistical values for the combined data set are listed in Table 3. For such a large sample

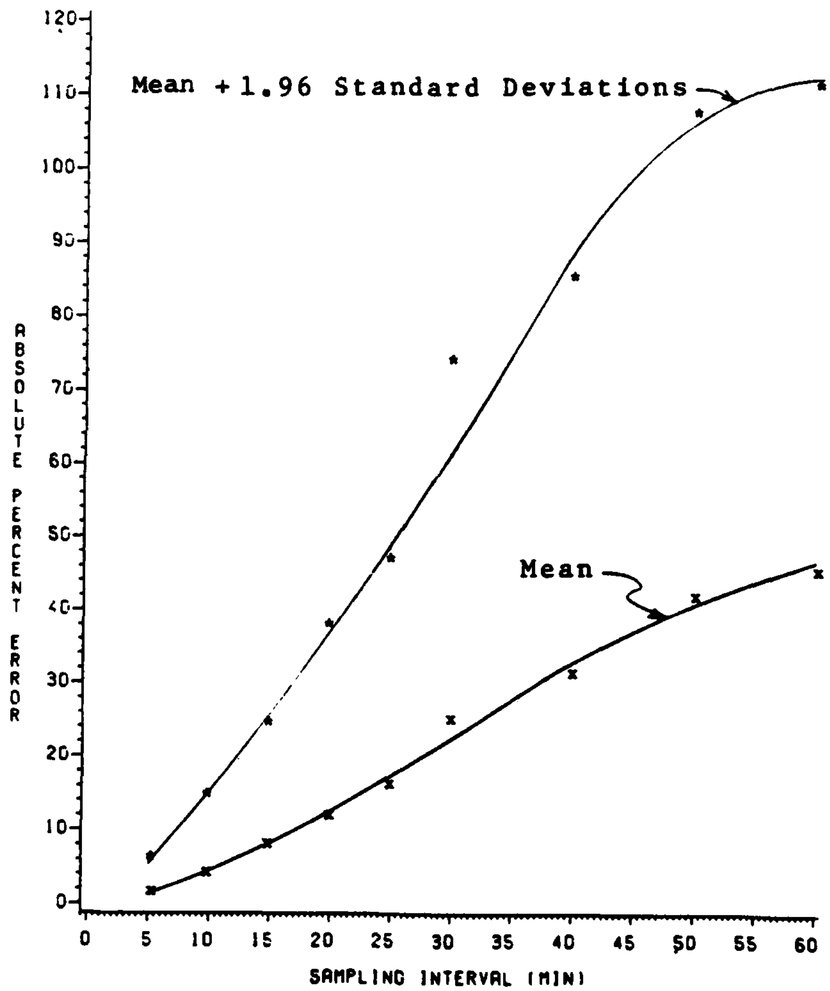


Fig. 6. Plot of mean absolute errors by sample interval. Data were the combined ADATA and BDATA sets, 192 observations per plotted point.

Table 3. Summary of the statistical values of absolute percent error by sample interval (min) for the combined data set.

Sample interval (min)	Minimum error (%)	Mean error (%)	Maximum error (%)	Standard deviation
		Data set = combined data		
5	0.00	1.45	32.31	2.49
10	0.00	4.45	26.22	5.04
15	0.00	8.02	51.33	8.37
20	0.07	12.28	131.44	13.14
25	0.00	16.21	82.60	15.62
30	0.10	24.93	177.43	25.32
40	0.66	31.61	149.18	27.60
50	0.24	41.99	181.39	33.95
60	0.30	45.44	254.88	34.33

size the errors are assumed to be normally distributed. Thus, it can be said that 95% of the absolute errors measured would be expected to lie within limits described as

$$95\% \text{ of the absolute errors} \leq \overline{APE} + 1.96\, s \quad , \tag{9}$$

where $\overline{APE}$ is the mean absolute percent error and s is the associated standard deviation. This limit, when plotted for each sample interval, results in the upper curve in Fig. 6. The curves were fit to the data with a SAS cubic regression drawing routine. Since the combination of data sets almost doubled the sample size this plot is probably more representative than either of the separate set's curves taken individually.

In Fig. 6 the mean errors, the lower curve, increase as expected with increasing sampling interval. The lower slope of the curve at

larger sample intervals may be due to the extrapolation technique used. It must be remembered that the 50 minute interval had 30 minutes of extrapolated data, whereas the 60 minute interval had only 20 minutes. Thus, somewhat less confidence can be placed in the curves beyond the 40 minute point on all plots that use the sample interval as the abscissa.

With the above stated cautions applied, Fig. 6 and Table 3 can be used to approximate mean errors from other large samples. The mean errors summarized here are very similar to those found by Wilson (1970) in a study of convective storm radar-derived rain total estimates. By comparison he found 45%, 25%, and 13% mean error at the 60, 30, and 15 minute sample intervals respectively. In Table 3 the combined data shows that these errors compare favorably with the 45%, 25%, and 8% mean errors found in this study. The upper curve in Fig. 6 defines the upper limit of the area within which the errors would fall for 95 out of 100 estimates with a specific sample interval. Nineth-five percent of the errors of total rain estimates should be less thant 50% if a sample interval of 25 minutes is used. To be within 25% error an interval of 15 minutes is necessary. To keep the absolute percent error of a total rain estimate less than 100% a maximum sample interval of approximately 46 minutes would be necessary. As a rough check on the statistical accuracy of the upper curve in Fig. 6, it was found that 96.01% of the measured errors fell within its limits. This fact reinforces the validity of the curve and our basic assumptions.

The combined data set's errors were also related to the number of samples taken in the 80 minute time span of observations. At each of

the discrete "number of samples" points (i.e., 2, 3, 4, 5, 6, 9, and 17) the mean errors and their standard deviations were calculated. The results are summarized in Table 4. The mean errors are plotted in Fig. 7, along with the limit curve defined previously in Eq. (9).

Table 4. Summary of statistical values of absolute percent error by number of samples per 80 min time span.

Number of samples	Minimum error (%)	Mean error (%)	Maximum error (%)	Standard deviation
		Data set = ADATA		
2	0.24	41.82	128.85	28.19
3	0.53	26.46	138.60	23.59
4	0.31	14.53	71.29	12.72
5	0.07	11.77	131.44	14.53
6	0.00	7.52	38.56	7.17
9	0.00	4.56	26.16	5.38
17	0.00	1.23	5.76	1.16
		Data set = BDATA		
2	0.27	45.92	254.88	39.91
3	0.10	30.36	177.43	29.75
4	0.00	18.14	82.60	18.30
5	0.23	12.87	52.56	11.39
6	0.00	8.61	51.33	9.58
9	0.00	4.31	26.22	4.65
17	0.00	1.70	32.21	3.43
		Data set = combined data		
2	0.24	43.72	254.88	34.14
3	0.o0	28.27	177.43	26.66
4	0.00	16.21	82.60	15.62
5	0.07	12.28	131.44	13.14
6	0.00	8.02	51.33	8.37
9	0.00	4.45	26.22	5.04
17	0.00	1.45	32.31	2.49

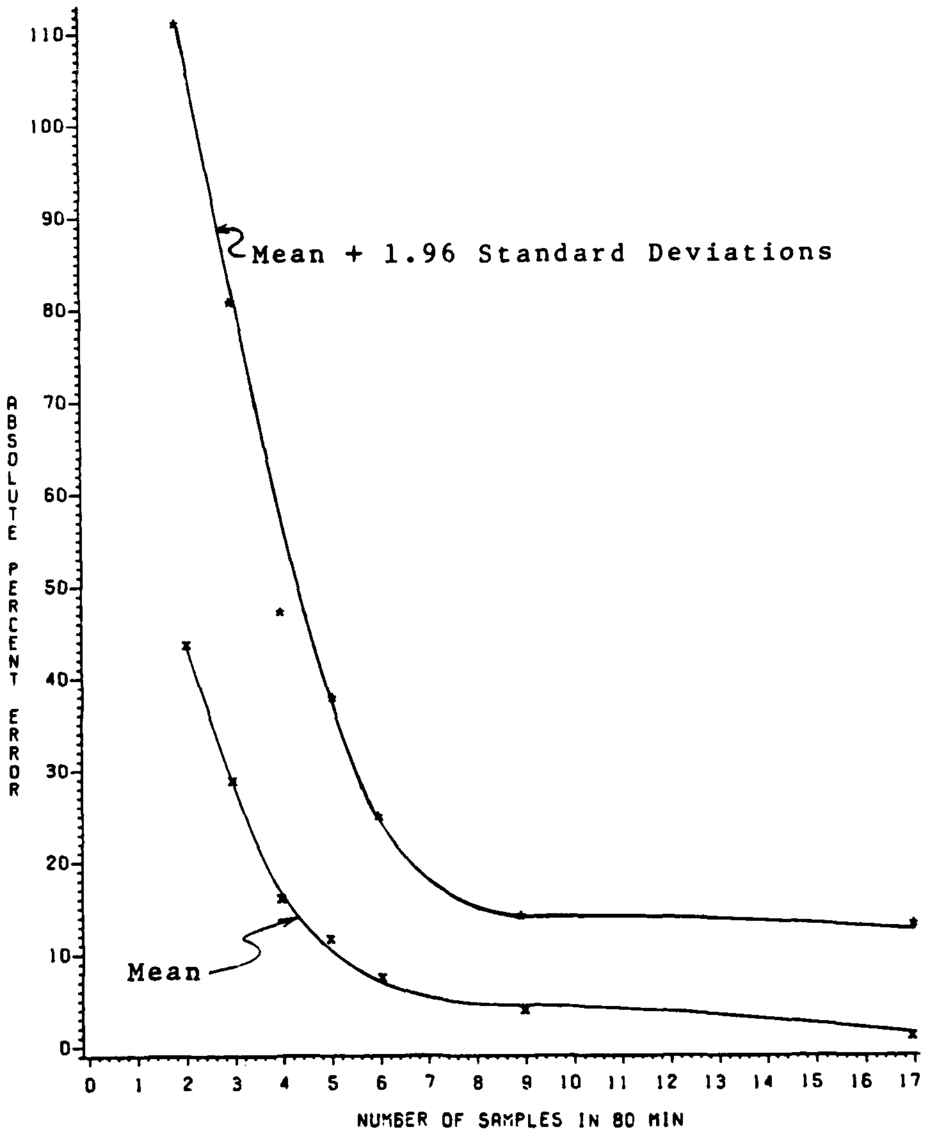


Fig. 7. Plot of mean absolute percent errors by the number of radar samples taken during the 80 min time span.

The plots show the striking lack of difference in errors for numbers of samples greater than eight. The mean error from this point and greater is less than approximately 5%. Evidently this is a point at which the rainfall temporal profile becomes fairly well defined. Increasing the number of samples beyond this number has little effect in decreasing the errors because the rain profile is almost as well defined as can be. The mean errors increase dramatically when less than six samples are taken. If only two samples are taken in an 80 minute span the error can be expected to be less than approximately 110% in 95 out of 100 cases. If an error of less than 50% or 25% is required the upper curve indicates that at least 4 or 6 samples, respectively, would be necessary during an 80 minute observation period.

Correlations and Analyses of Variance

Previous investigators have been cited that established several variables important to the explanation of errors of total rain estimates. This present study has beasured or calculated values for the independent variables of total rain, mean rain rate, sequential variability, storm depth, storm speed, and storm sampled depth for each grid area. The other variables are, of course, the sample interval with which the storm is observed and the number of samples taken of the storm.

This section deals with finding the relative importance of each of the variables in relation to the errors of rainfall estimates. The next step is categorizing or subdividing the variables into groups to further delineate the errors.

Which variables have a significant effect or are highly correlated with the measured errors? A correlation analysis (SAS Institute Inc., 1982b) was accomplished to determine the relations of the independent variables to the mean errors. These analyses were done only on the ADATA set because of inclusion of the mean rain rate variable. The log of the sample interval and the log of the number of samples clearly had the highest correlation coefficients, .73 and -.73 respectively, with the log of the absolute percent errors. The next highest correlation coefficient with the error was .21, with the sequential variability. One-minute rain totals and mean rain rates have similar correlation coefficients of .12. The last three variables, the precipitation area characteristics, are all slightly negatively correlated to the error, with correlation coefficients from -.02 to -.05.

It is obvious that the sample interval selected and the number of samples taken would have the most effect on the rain estimate's errors. The other variables can not be dismissed outright as unimportant because of the wide ranges of the measured variables. The sequential variability varied by a factor of nearly 50 in its untransformed state, while the others varied from a factor of 5 to over 25. It may be that each variable is more or possibly less correlated with the mean errors depending on where in the ranges the measurements were made. For example, were the large mean errors associated with large rain rates, or faster moving storm areas? Even though this study only included storms of a cold-front type, Huff (1970) found great variability within storms of the same synoptic type.

In view of this variability it was decided to categorize the

independent variables in an attempt to further investigate the factors to which the sampling errors could be attributed. Disregarding the variables of sampling technique, the mean rain rate of a storm area is the only one of the next three largest correlations that is roughly estimable with one or two radar scans, so it was a candidate for subdivision into categories. The area sampled depth and the area speed were found to be highly correlated with a correlation coefficient of .992. This was known beforehand because the speed was used to calculate the sampled depth. The speed was the selected variable here because it could be determined more directly with at least two radar scans of an area. The storm depth was also categorized because it was not highly correlated with any of the other variables.

The subdivision of the selected variables was done subjectively by investigating the ranges of each. The variables were categorized into three subdivisions each. The results of these analyses were plotted using a SAS cubic regression routine to draw the lines-of-best-fit.

The mean rainfall rate was divided into rates of less than 2.4 mm/h (light), with 2.4 and 12 mm/h (medium), and greater than 12 mm/h (intense). There were 180 observations in the light class, 630 in the medium class, and 114 in the intense rain rate class. The mean errors for each subdivision were calculated for each sample interval. These calculations had 20 observations per mean error in the light class, 70 in the medium class, and 13 in the intense class. The results were plotted and are shown in Fig. 8. The outstanding difference in the figure is that the light rain rate class appears to have considerable less error than the other classes at large sample intervals. Statistical

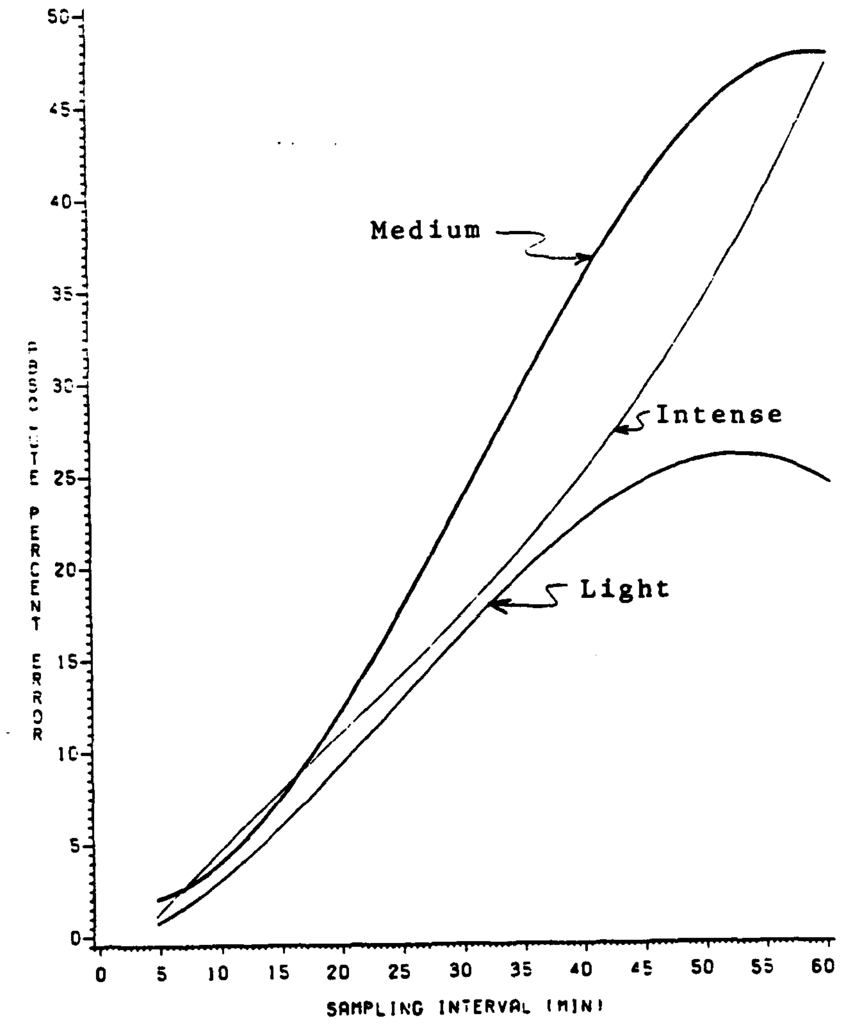


Fig. 8. Plot of mean absolute error by sample interval for categories of mean rain rate (mm/h).

testing was done to see if the classes differed significantly.

An analysis of variance was done to compare the mean log transformed errors between the subdivisions of rain rates. This test calculated an F value of 7.85 and a level of significance of $p = .0004$. This pointed to a significant difference between the mean errors of at least two classes of rain rates. Fisher's least significant difference t-test was applied to test which means differed. The results showed the overall mean errors of the light rate class differed significantly and were less than either of the other classes. The more conservative Tukey's studentized range test found the light rain rate significantly less than only the medium rate.

To further pinpoint the rain rate class differences another analysis of variance was run on the data by sample intervals. If this test indicated significant differences between rates of classes for a certain sample interval then Fisher's and Tukey's tests were also run on the data. The analysis of variance indicated significant differences for only the 5, 20, and 60 minute intervals.

With a sample interval of 5 minutes the analysis calculated an F value of 7.00 and a level of significance of $p = .0014$, indicating a significant difference. Fisher's test then showed that the medium rate errors differed from the light and intense rate categories. Tukey's test found the medium rate errors to differ only from the light rates. Since in this interval t: mean error was less than 1.5% and the greatest single error was less than 6%. These differences for the 5 minute interval are of minor importance.

The analysis of variance on the 20 minute sample interval

calculated an F value of 4.36 and a level of significance of p = .0152, indicating significant differences. With Fisher's and Tukey's tests on this interval the light rate errors were found to be significantly less than the medium and intense rates.

Finally, an analysis of variance run on the 60 minute sample interval calculated an F value of 5.86 and a level of significance of p = .0039, again indicating a significant difference in mean errors of the rain rate classes. Further testing with Fisher's test showed the light rain rate errors differ from the medium and intense rate errors. Tukey's test showed that the light rate errors differ only from the medium rate errors.

In summary the light rain rate class had several significant differences. In Fig. 8 it appears that the light rain rate class had less mean error than the other classes at every interval. The analysis of variance performed on the mean errors of these subdivisions found the light rain rate to be significantly less than the other classes. Yet, when the light rain rate class was further tested by sample interval it was found to be significantly less in only the 20 and 60 minute intervals. There is an imbalance in sample sizes in the rain rate classes by sample interval as the light rain rate class mean was of 20 observations, the medium mean was of 70, and the intense mean was of only 13. If this same analysis was run on a more balanced class structure, each with at least 30 observations, the results would be more conclusive. As it stands the tests found only two out of nine sample interval's mean errors in the light rain rate class and the overall class mean error to be significantly less than the other classes. With sample size kept in

mind it can not be concluded that the difference in the light rain rate class is of overall significance.

The next classifications of the data were by precipitation area depths. Fig. 9 is a plot of the mean absolute percent error against the sample intervals for the three subdivisions of area depth. The classes were by precipitation area depths greater than 80 km (wide), depths within 40 km to 80 km (medium), and depths of less than 40 km (thin). There were 351 observations in the wide class, 360 in the medium class, and 216 in the thin class. The analysis of variance performed on the mean errors of these classes resulted in an F value of 1.78 with a level of significance of $p = .1699$. This indicated no significant differences between the mean errors of these precipitation area depth classes. In Fig. 9 the mean errors show no large differences for any class at any specified interval. In the plot there is a mild overall tendency for the thin depths to have greater errors and the wide depths to have less. Synoptically this probably relates to the fact that more turbulent activity may often be associated with a smaller, thinner line type of precipitation event and more steady, stratiform type rain associated with a wider depth storm area. Overall, the statistical testing did not show these depth class error differences to be significant. Evidently classifications by storm depth do not result in the greater definition of the mean errors of total rain estimates.

The last subdivisions of the data were by the precipitation area's speed. The divisions were for area's speed greater than 50 km/h (fast), for speeds within 30 km/h to 50 km/h (medium), and for speeds less than 30 km/h (slow). This divided the total number of observations into

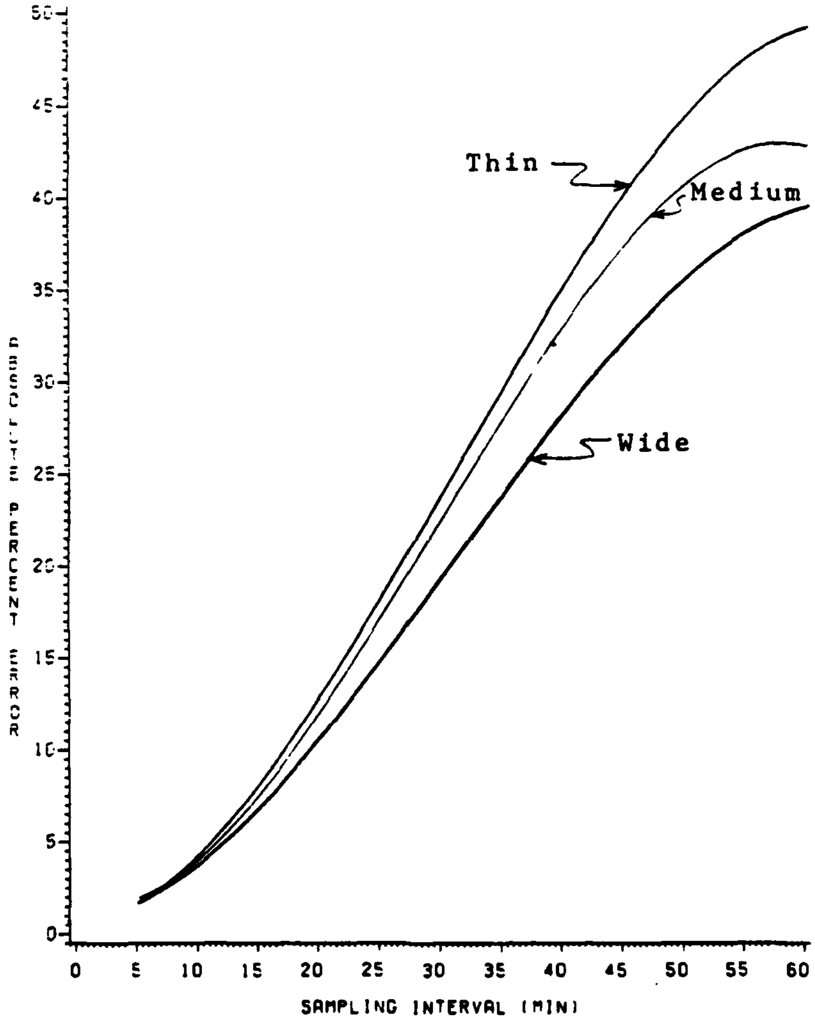


Fig. 9. Plot of mean absolute error by sample interval for categories of precipitation area depth (km).

groups of 90 in the fast class, 423 in the medium class, and 414 in the slow class. The analysis of variance performed on mean errors by class produced an F value of .250 with a level of significance of p = .7775. This result indicated no significant differences between class mean errors for the speed subdivisions. The mean absolute percent errors of these classes were plotted against the sample intervals in Fig. 10. In the figure the main difference in the respective curves is that the fast class, at longer sample intervals has less mean error than the other classes. This may be due indirectly to the imbalance of class sizes. This small sample of fast moving areas could be unrepresentative of the population. The fast class had less than one-fourth the observations of the other classes. For this reason there can be less confidence placed in conclusions about the fast class. It appears that subdividing rainfall data into area speeds does nothing to help explain more of the mean errors of total rain estimates.

Regression Analyses

Regression analyses are tools for further examining and even predicting the mean errors associated with different sampling measures. A correlation analysis is a starting point for selecting possible variables to include in a regression. Since mean rain rate was a variable of interest the analysis was done using the ADATA data. Error and rainfall measurements were log transformed to approach normalization. It is obvious, from the correlations previously stated, that the sample interval, the number of samples, the sequential variability, and the rainfall measurements were important in explaining the errors of total rain

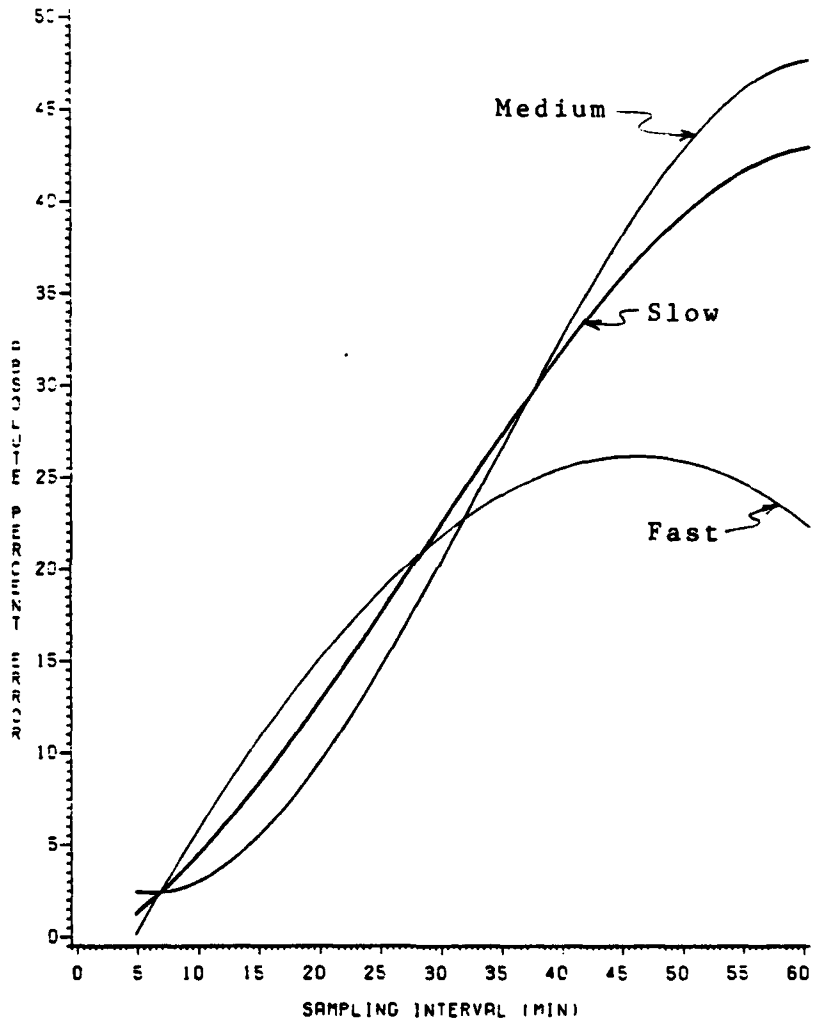


Fig. 10. Plot of mean absolute error by sample interval for categories of precipitation area speed (km/h).

estimates. There was little correlation between the precipitation area's physical characteristics of speed or depth, and the errors. The higher correlations of the sample intervals and numbers of samples to the errors point to their greater importance in a regression relation.

In addition to the eight previously explained variables that were discussed earlier, it was felt that several other variable products could have been important. Variables that could be estimated with two or less radar scans would be useful in a predictive regression equation. The sample interval, area depth, area speed, and mean rain rate were candidates for the variable products. Since these variables could be determined or estimated before a storm was over the area of interest, these variables were of use in a predictive sense. Included in the regressions were products of the sample interval with the speed, rain rate, and area depth, products of the rain rate with the speed and depth, and the product of the speed and depth.

The next step was to do simple linear regressions with a procedure, RSQUARE (SAS Institute Inc., 1982a), that does all possible regressions between the different conbinations of the independent variables and given variables. The ADATA data were used because of their valid rain rate measurements. The procedure was run first on the variables, than on their natural log transforms, and finally on the variable products. Table 5 summarized the results of the variable's association with the log of the absolute percent error.

In Table 5 the multiple correlation coefficients, R, are an expression of the degree of association between the variables in the model. The coefficient of determination, R^2, when multiplied by 100 is

Table 5. Coefficients of determination and correlation coefficients for variable's association with the log of the absolute percent error as the dependent variable.

Independent variables	R^2	R
Area speed, SPD	.0006	.0245
Sampled depth, SAMPDEP	.0007	.0265
Area depth, STRMDEP	.0025	.0500
Total rain, R_t	.0056	.0748
Mean rainrate, R_a	.0085	.0922
Sequential variability, D	.0308	.1755
Number of samples, N	.4548	.6744
Sample interval, T	.4760	.6899
Ln (Mean rain rate), LRA	.0150	.1225
Ln (Total rain), LRT	.0153	.1237
Ln (Seq. variability), LD	.0460	.2145
Ln (Number of samples), LN	.5273	.7262
Ln (Sample interval), LT	.5286	.7270
SPD X STRMDEP	.0007	.0265
Ln (R_a X SPD)	.0074	.0860
Ln (R_a X STRMDEP)	.0074	.0860
T X SPD	.2353	.4851
T X STRMDEP	.2790	.5282
Ln (R_a X T)	.3520	.5933

the percent of variance in the dependent variable that can be explained by the independent variable or model. The table clearly shows the sample interval and the number of samples taken to be the most important variables in any regression used to predict errors. The natural log transformed values of these variables have greater R^2 values than do the untransformed values, thus the transformed values should be used in a final regression model.

The sequential variability is seen as the next important variable for explaining the errors. Its transformed value shows a R of 0.2145. This evidently makes it a more important value than either the rainfall

variables or their transforms. This differs from Mueller's (1957) findings that related the standard error of raingage estimates to the same type variables used in this study. He concluded that the best relationship existed between the standard error of the rainfall estimate with the sample interval and the total rainfall, instead of with the sample interval and the sequential variability. Mueller's total rain was for the entire storm time, which is significantly different from the total rain in this study. Here the total rain was measured for only 80 minutes, which, may or more often may not have been the entire storm time. This also means that in this study rain along the storm's leading edge was more often measured. The leading edge of a cold-front may structurally be more active and turbulent, resulting in a more erratic temporal rain profile and thus a higher sequential variability with generally larger sampling errors. This may account for the greater relative importance of the sequential variability in this study.

The mean rain rate and the total rain appear to be of relatively less importance in Table 5, as indicated by their R^2 values. They explain a maximum of only 1.5% of the error's variability even when transformed. These variables were not considered important enough to be included in the final regression model.

The precipitation area's physical characteristics of speed, depth and sampled depth have very small R^2 values. These measurements are then nearly useless in helping to explain the errors in total rainfall estimates. For this reason these variables were not log transformed for further work in obtaining the final regression equation.

Table 5 has very small R^2 values for all of the attempted variable

products. All products involving the sample interval have much smaller R^2 values than do the sample interval by itself. Since the products and the interval are mutually exclusive in a regression, only the sample interval should be used. The remaining products of speed, depth and rain rate explain less than a maximum of 0.7% of the error variability. Therefore, they are of no use in a final regression equation.

Table 6 shows the complete result of all possible regressions on the log transformed model with the relatively important variables. The sample interval and the number of samples are just two methods of quantifying the radar sampling of a storm, therefore they are mutually exclusive in a regression. This means that there is a "best" model for each of these variables. In Table 6 the log of either sampling method alone accounts for approximately 53% of the error variability. Adding the sequential variability to either sampling method adds less than 5% to the explained variability. The addition of either rainfall variable to the model with sampling method and sequential variability increases the R^2 by nearly 7 percent. Including both rainfall parameters in that model results in approximately a 9 percent increase in explained variance of the errors.

These results were then used to construct the "best" regression models. It was immediately obvious that the sample interval and the number of samples were the variables of most importance and, a separate model would have to be made for each. The two resulting equations would allow prediction of the sampling errors by determining a priori either the sampling interval desired or the number of samples possible in a specified storm. The sequential variability and the total rainfall were

Table 6. All possible regressions on dependent variable LAPE, natural log of the absolute percent error.

Number in model	R^2	Variables in model
1	0.015	LRA
1	0.015	LRT
1	0.046	LD
1	0.527	LN
1	0.529	LT
2	0.110	LD LRA
2	0.191	LRT LRA
2	0.244	AD LRT
2	0.527	LN LRT
2	0.529	LT LRT
2	0.530	LT LN
2	0.542	LN LRA
2	0.544	LT LRA
2	0.573	LN LD
2	0.575	LT LD
3	0.244	LD LRT LRA
3	0.530	LT LN LRT
3	0.545	LT LN LRA
3	0.576	LT LN LD
3	0.586	LT LRT LRA
3	0.587	LN LRT LRA
3	0.637	LN LD LRA
3	0.638	LT LD LRA
3	0.646	LT LD LRT
3	0.646	LN LD LRT
4	0.588	LT LN LRT LRA
4	0.640	LT LN LD LRA
4	0.647	LT LN LD LRT
4	0.659	LT LD LRT LRA
4	0.660	LN LD LRT LRA
5	0.661	LT LN LD LRT LRA

seen to be somewhat important but are only known after the fact. They should be included in a type of equation for post event analyses but they are not useful in the predictive sense. Since a main objective of

this study was to develop predictive regression equations it was decided to not include the sequential variability or the total rain in further modeling. The mean rain rate can be roughly estimated with one or two radar scans, therefore would be useful in a predictive equation. In Table 6 the addition of the mean rain rate to either sampling model only resulted in an increase of approximately 0.015 in R^2. This slight amount of improvement in the model was by far outweighed by the fact that radar scans had to be made for even a rough estimate of the rain rate. For this reason the rain rate was not included in the final model, which then allowed estimation of errors without turning on the radar.

The two "best" regression models for the stated objectives were then of the simple single variable type. One model,

$$LAPE = a + b\,(LT) \quad , \tag{10}$$

relates the natural log of the absolute percent error (LAPE) and the natural log of the sample interval (LT) in minutes, with a and b regression constants. The other model's equation is

$$LAPE = a + b\,(LN) \quad , \tag{11}$$

which relates the log of the absolute percent error (LAPE) to the log of the number of samples taken (LN), again with a and b regression constants. These single variable regression models have a R^2 value of approximately 0.53, as seen in Table 6. The advantage to such simple models is that they require no measurement of storm characteristics from preliminary radar scans prior to the start of rainfall measurements.

Final regressions on the models were done with the combined data sets. In Figs. 6 and 7 the data indicated that a better fit might be found using a second or third order polynomial regression. The General Linear Models (GLM) regression procedure (SAS Institute Inc., 1982b) was executed with three models for each of the two independent variables. The models were

$$\text{Linear} \qquad LnY = a + b(LnX) \tag{12}$$

$$\text{Quadratic} \qquad LnY = a + b(LnX) + c(LnX)^2 \tag{13}$$

$$\text{Cubic} \qquad LnY = a + b(LnX) + c(LnX)^2 + d(LnX)^3 \tag{14}$$

where Y is the dependent variable, absolute percent error plus 2 (to avoid Ln(Ø) computations), a, b, c, and d are regression constants, and X is the independent variable. The variable, X, is the sample interval in one model, then the number of samples in the other.

Regression analysis was done first using the log of the sample interval (LT) in minutes as the independent variable in Eqs. (12), (13), and (14). The results are in the Appendix in Tables 9, 10, and 11 respectively. There was no significant improvement in R^2 for the polynomial models. The quadratic parameter had a p-value of 0.2131 which makes it not significant to the regression. In the cubic regression the intercept and linear parameters were insignificant to the regression and higher order terms, LT^2 and LT^3, were significant with p=values of 0.0319 and 0.0392 respectively. The polynomials showed increased standatd errors of estimates over the linear model. The overall results indicate the best regression is the linear model

$$APE = e^{-1.212}\ T^{1.161} - 2 \tag{15}$$

where APE is the absolute percent error and T is the sample interval in minutes. Here, 2 is subtracted because it was added previously to the regression model to avoid Ln(Ø) computations. This model's predicted error values are plotted with the actual observed mean errors in Fig. 11. As the sample interval increases the model underestimates of the mean error increases significantly. The model underestimates the observed mean errors by over 20% beyond sample intervals of 25 minutes. The underestimation is a bias that is a consequence of using the log transforms for regressions. The bias makes Eq. (15) useful for prediction of errors only at sample intervals below approximately 15 minutes.

Freund (1977) states a method which is used to reduce the bias when working with log transformed data. A bias correction factor can be added before the antilogarithms of the estimated errors are taken. The equation for this correction in this case becomes

$$APE^* = Ln^{-1}\,[(LAPE) + k(MSE\ model)]\ -2 \tag{16}$$

where APE^* is the bias corrected absolute percent error, LAPE is the log of the absolute percent error calculated by the regression equation, k = 0.5, and MSE is the mean square error of the regression model. With the appropriate numbers substituted this equation becomes

$$APE^* = Ln^{-1}\,[(-1.212+1.161\ LT) + 0.5\ (0.761)]\ -2 \tag{17}$$

where LT is the log of the sample interval in minutes. This equation can be written in a multiplicative form as

$$APE^* = e^{-.832}\ T^{1.161}\ -2 \tag{18}$$

where T is the sample interval in minutes.

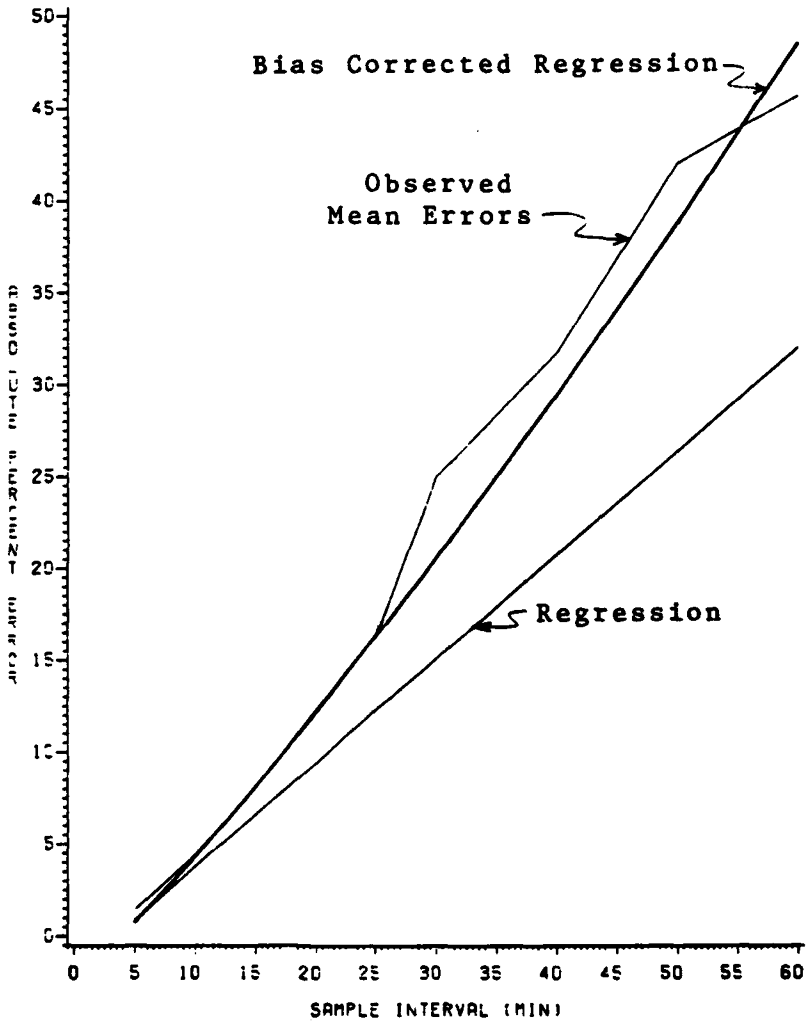


Fig. 11. Plot of regression predicted absolute percent error by sample interval. Observed mean errors, a linear regression line-of-best-fit, and the linear regression line when corrected for log bias are shown.

The bias corrected absolute percent errors were calculated using Eq. (18). The results are listed in Table 7 and plotted in Fig. 11. In Fig. 11 the corrected errors are seen to closely approximate the observed errors. Although the regression equation estimates errors that are more in agreement with the actual observed errors, it must be stated that the bias correction technique is not mathematically exact enough to eliminate totally all bias errors. With that caution in mind useful prediction limits can be drawn about the regression line.

Table 7. Summary, by sample interval, of regression errors, bias corrected regression errors, and upper 95% prediction limits.

Sample interval (min)	Observed errors (%)	Regression predicted errors (%)	Corrected predicted errors (%)	95% prediction limits (%)
5	1.4	- .1	.8	3.4
10	4.4	2.3	4.3	18.6
15	8.0	4.9	8.1	34.0
20	12.3	7.6	12.1	50.8
25	16.2	10.5	16.3	68.4
30	24.9	13.4	20.6	86.5
40	31.6	19.6	29.5	123.9
50	42.0	25.9	38.8	163.0
60	45.4	32.5	48.5	203.8

Fig. 12 is a plot of the bias corrected regression estimated errors by sample interval, with the observed errors as points. The upper 95% prediction limit was calculated at each sample interval point using a standard equation from Koopmans (1981). The prediction limit values for each sample interval are listed in Table 7. The width of the prediction limit reflects the great variability in the sample measurements,

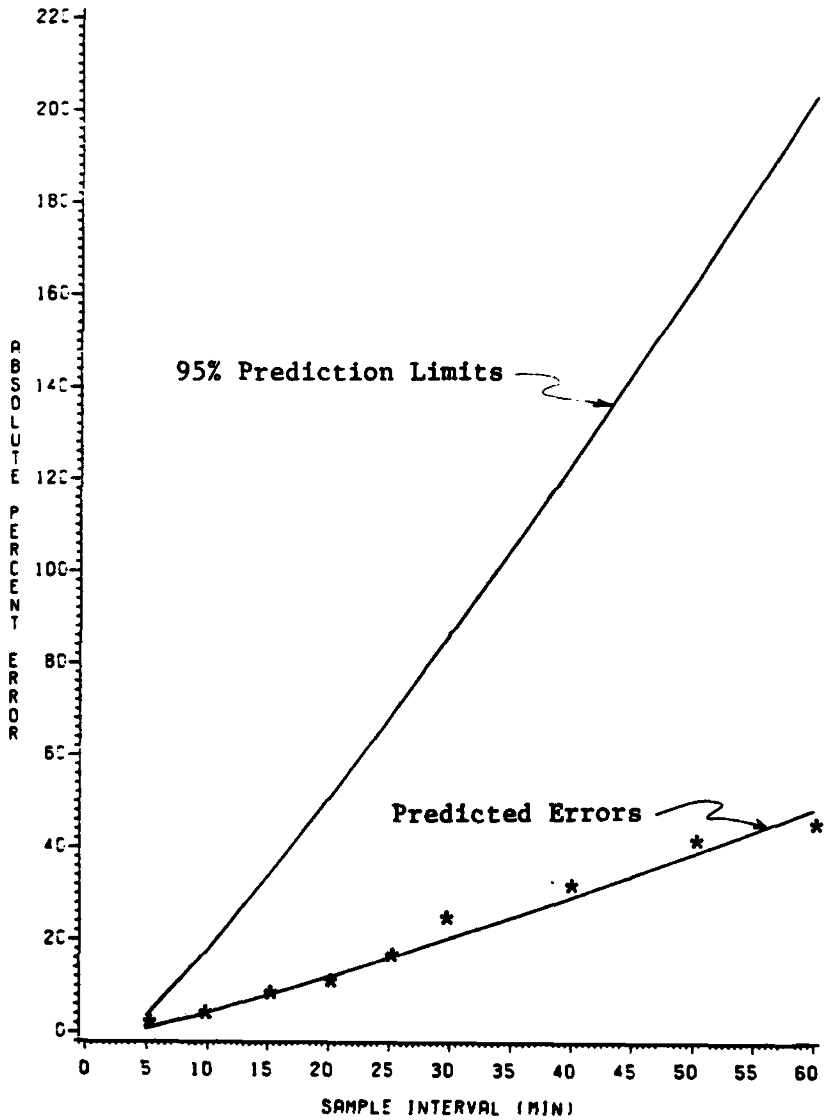


Fig. 12. Plot of the bias corrected regression predicted error with the upper 95% prediction limits by sample intervals. Mean observed errors for each sample interval are plotted as "*".

particularly at the larger sample intervals. This is the limit that the errors would be expected to be within 95% of the time for any single grid area sample. This prediction limit then clearly illustrates the wide error range to be expected when making total rainfall estimations for small numbers of samples or small areas. For example, to be assured of less than 100% error (95% of the time) the sample interval must be less than approximately 34 minutes. Conversely, a sample interval of 20 minutes would result in total rain estimation errors of less than 50%, in 95 out of 100 cases.

The log of the number of samples (LN) was next used as the independent variable in Eqs. (12), (13), and (14). The results of the three regressions are in the Appendix in Tables 12, 13, and 14 respectively. The linear model gives a fairly good fit to the data. The highly significant p-values, less than 0.0001, and the small standard errors are evidence of a good regression fit.

There was insignificant improvement in R^2 for the higher order polynomials. The linear model's $R^2 = 0.498$, the quadratic $R^2 = 0.499$, and the cubic $R^2 = 0.500$. The quadratic parameter LN^2 was not significant ($p = .0508$), and the standard errors increased. Thus the quadratic model's fit was not as good overall as the linear's. The cubic model results indicated an even worse fit. The linear parameter LN was insignificant ($p = .9062$), and again the higher order terms were significant. Their p-values indicate that a t-statistic this large or larger could have been found by chance alone in nearly 3 and 5 cases out of 100, for the LN^2 and LN^3 parameters respectively.

The linear regression model is the best in this set. It can be

written with regression constants as

$$APE = e^{4.339} N^{-1.309} - 2 \tag{19}$$

where N is the number of samples upon which the total rain estimate was based, and APE is the absolute percent error.

A plot of the predicted values from this model with the actual observed mean errors is seen in Fig. 13. Obviously the same bias effects are seen here, again due to the log transformations used in regressions. The predicted errors range from 20% less than the actual sample errors, when taking more than approximately nine samples, to over 60% less with two or three samples taken. This leaves the important, small number of sample's errors relatively unpredictable when using this regression equation.

The bias correction, Eq. (16), was used with Eq. (19) resulting in a corrected form

$$APE^{*} = Ln^{-1} [(4.339 - 1.309 \, LN) + 0.5 \, (0.7676)] - 2 \tag{20}$$

where LN is the log of the number of samples. This equation can also be written in a multiplicative form as

$$APE^{*} = e^{4.723} N^{-1.309} - 2 \tag{21}$$

where N is the number of samples taken in 80 minutes.

Using Eq. (21), values of the bias corrected regression estimated errors were calculated, then listed in Table 8 and plotted in Fig. 13. The bias corrected regression estimates are a very near approximation to the observed errors, as seen in Fig. 13. This regression relation can

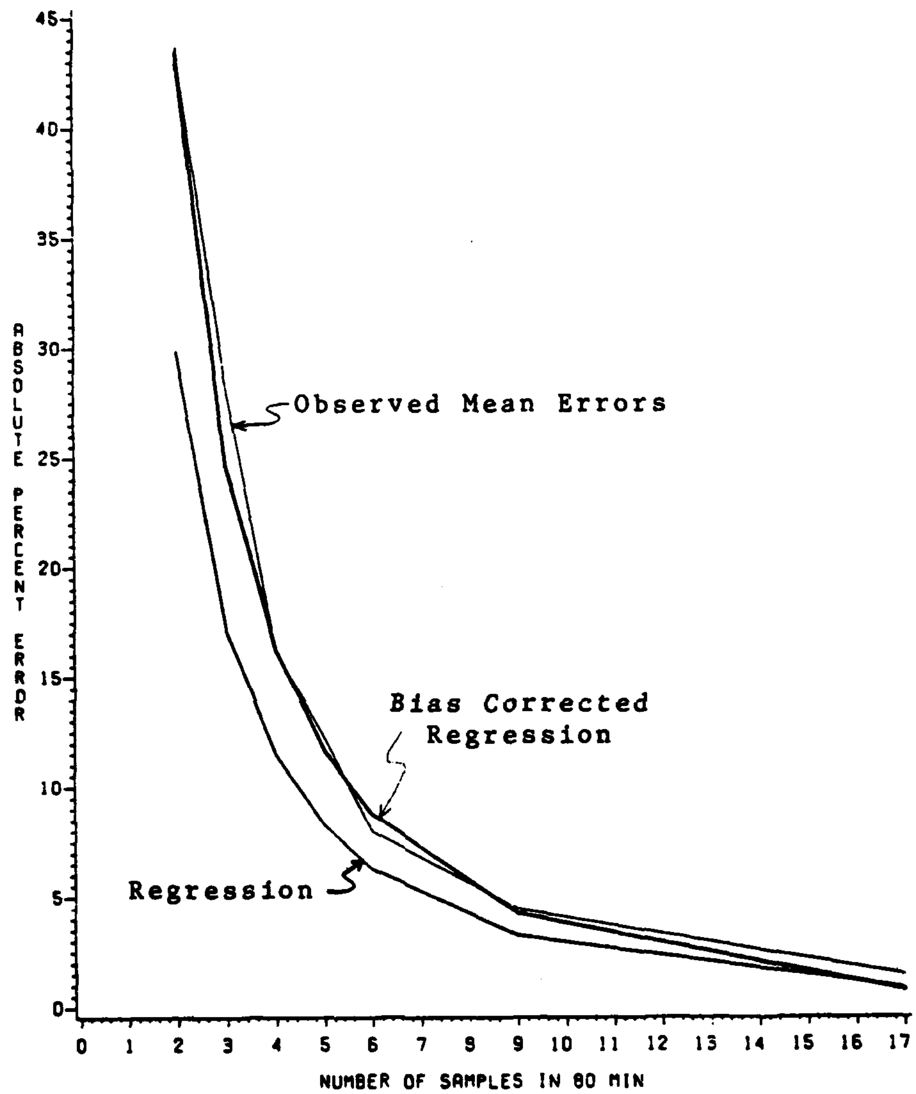


Fig. 13. Plot of regression predicted absolute percent error by number of samples. Observed mean errors, regression line-of-best-fit, and the corrected regression line-of-best-fit are shown.

Table 8. Summary, by numbers of samples, of regression errors, bias corrected regression errors, and upper 95% prediction limits.

Number of samples	Observed errors (%)	Regression predicted errors (%)	Corrected predicted errors (%)	Upper 95% prediction limits (%)
2	43.7	28.9	43.4	183.8
3	28.3	16.2	24.7	104.4
4	16.2	10.5	16.3	68.9
5	12.3	7.3	11.7	49.5
6	8.0	5.3	8.8	37.2
9	4.4	2.3	4.3	18.2
17	1.4	- 0.1	0.8	3.2

then be used to plot useful prediction limits.

Fig. 14 is a plot of the bias corrected estimated errors with their associated upper 95% prediction limit plotted by the number of samples taken. The calculated values are listed in Table 8. The plot is of the same general form as the empirically derived Fig. 8. The prediction limit allows estimation of the largest expected error of any single total rain sample. For example, the error would be expected to be less than 50% (95% of the time) if at least five samples were taken in an 80 minute time span. The figure also illustrates the problem of over-sampling, which is actually the lack of significant improvement of the accuracy of estimates made with more than nine samples during the 80 minutes.

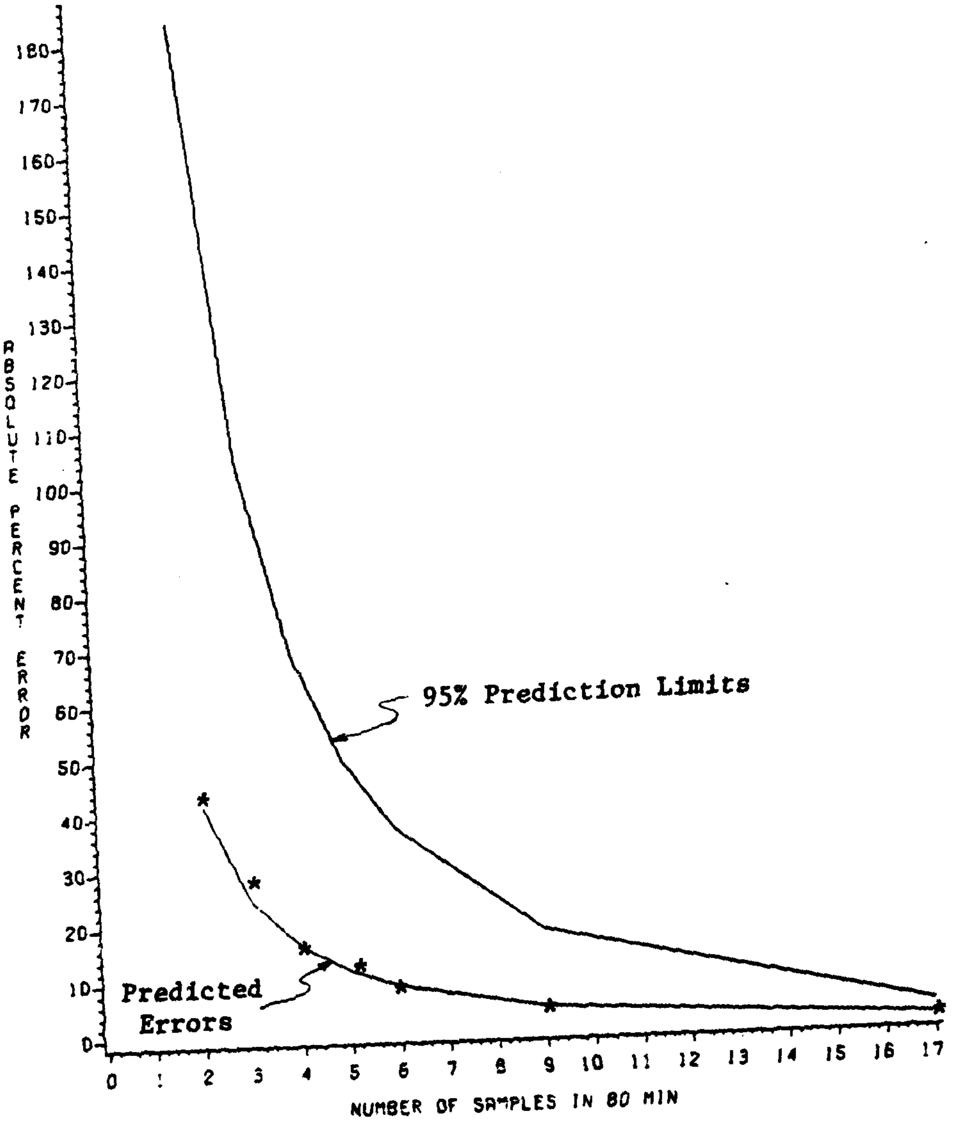


Fig. 14. Plot of bias corrected regression predicted errors with the upper 95% prediction limits by the number of samples taken in 80 min. Mean observed errors at number of samples points are plotted as "*".

CHAPTER IV

SUMMARY AND CONCLUSIONS

This study investigated the effects of various sampling-rates on radar-derived, total rainfall estimates. Radar observations, taken at one minute intervals, were recorded for nine storms in 1984. Total rainfall estimates, for 10 km by 10 km areas, based on these data were considered "ground truth" totals. Sample-rates, ranging from 5 to 60 min, were applied to the recorded data to calculate total rain estimates for each sample rate. These derived rain totals were compared with the "ground truth" totals, with the differences referred to as "errors." These errors were plotted against the sampling-rate and the number of samples taken. Other variables, investigated for high correlations with the errors, were the mean rain rate, total rain, sequential variability, storm width, and storm speed of movement. Analyses of variance were done on subdivisions of the storm width, storm speed, and mean rain rate variables. Regression analyses determined the "best" models, using error as the dependent variable, to allow prediction of the expected errors in total rain estimates.

With this study's sample size in mind, several conclusions can be drawn:

(1) Large-sample mean absolute errors of total rain estimates ranged from nearly 8%, 25%, and 45% with 15, 30, and 60 min sample-rates, respectively.

(2) 95% of the individual estimate errors were found to be less than approximately 25%, 75%, and 112% with 15, 30, and 60 min

sample-rates, respectively.

(3) Taking more than 8 samples per 80 min period does not increase significantly the accuracy of the measurement. The mean error for greater than 8 samples taken was less than approximately 5%.

(4) If only 2, 4, or 6 samples are taken in 80 min, the error is espected to be less than 110%, 50%, or 25%, respectively, 95% of the time.

(5) The variables of highest correlation with the errors were the sample-rate and the number of samples taken, with coefficients of 0.73 and -0.73 respectively.

(6) The variables of sequential variability, mean rain rate, total rain, storm width, storm speed, and sampled storm depth had low correlations with the errors.

(7) Subdivisions of the variables were inconclusive because of small, unbalanced sample sizes.

(8) Regression equations were derived to relate the errors to the sample-rate and the number of samples taken. This allows prediction of errors for individual total rain estimates.

REFERENCES

Brandes, E. A., 1975: Optimizing rainfall estimates with the aid of radar. *J. Appl. Meteor.*, *14*, 1339-1345.

Freund, R. J., 1977: *Regression Methods*. Marcel Dekker Inc., New York, NY, 357 pp.

_____, 1981: *SAS for Linear Models*. SAS Institute Inc., Cary, NC, 231 pp.

Huff, F. A., 1970: Sampling errors in measurement of mean precipitation. *J. Appl. Meteor.*, *9*, 35-44.

_____, and J. C. Neill, 1957: Rainfall relations on small areas in Illinois. Illinois State Water Survey, Urbana, IL, Bulletin 46, 43-50.

Jatila, E., and T. Puhakka, 1973a: Experiments on the measurement of areal precipitation by radar. *Geophysica*, *12*, 103-125.

_____, and _____, 1973b: On the accuracy of radar rainfall measurements. *Geophysica*, *12*, 127-140.

Koopmans, L. H., 1981: *An Introduction to Contemporary Statistics*. Duxbury Press, Boston, MA, 599 pp.

Larson, L. W., 1974: Accuracy of precipitation measurements for hydrologic modeling. *Water Resour. Res.*, *10*, 857-863.

Linsley, R. K., and M. A. Kohler, 1951: Variations in storm rainfall over small areas. *Trans. Am. Geophys. Union*, *32*, 245-250.

McGuiness, J. L., 1963: Accuracy of estimating watershed mean rainfall. *J. Geophys. Res.*, *68*, 4763-4767.

Mueller, E. A., 1957: Study on intensity of surface precipitation using radar instrumentation. Illinois State Water Survey, Urbana, IL, Ninth Quarterly Tech. Report, 35 pp.

Niell, J. C., 1953: Analysis of 1952 radar and raingage data. Illinois State Water Survey, Urbana, IL, Research Report No. 2, 18 pp.

Ostle, B., and R. W. Mensing, 1982: *Statistics in Research*, Third Edition. Iowa State Univ. Press, Ames, Iowa, 688 pp.

SAS Institute Inc., 1982a: *SAS User's Guide: Statistics*, 1982 Edition. SAS Institute Inc., Cary, NC, 584 pp.

_____, 1982b: *SAS User's Guide*: *Basics*, 1982 Edition. SAS Institute Inc., Cary, NC, 923 pp.

Wilson, J. W., 1970: Operational Measurement of Rainfall with the WSR-57: Review and Recommendations. *Preprints 14th Radar Met. Conf.*, Tucson, AZ, Amer. Meteor. Soc., 126-168.

APPENDIX

Table 9. Regression results on the linear model LAPE = a + b(LT). LAPE is the natural log of (absolute percent errir + 2), a and b are regression coefficients, and LT is the natural log of the sample interval in minutes.

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: LAPE LOG (ABSOLUTE PERCENT ERROR+1)

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	1	1325.46786969	1325.46786969	1741.89
ERROR	1726	1313.37782483	0.76093733	PR > F
CORRECTED TOTAL	1727	2638.84569452		0.0001

R-SQUARE	C.V.	ROOT MSE	LAPE MEAN
0.502291	36.4886	0.87231722	2.39065575

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LT	1	1325.46786969	1741.89	0.0001

SOURCE	DF	TYPE III SS	F VALUE	PR > F
LT	1	1325.46786969	1741.89	0.0001

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	-1.21239178	-13.65	0.0001	0.08884346
LT	1.16097819	41.74	0.0001	0.02781723

Table 10. Regression results on the quadratic model LAPE = $a + b(LT) + c(LT)^2$.

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: LAPE LOG (ABSOLUTE PERCENT ERROR+1)

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	1326.64815755	663.32407878	872.00
ERROR	1725	1312.19753697	0.76069422	PR > F
CORRECTED TOTAL	1727	2638.84569452		0.0001

R-SQUARE	C.V.	ROOT MSE	LAPE MEAN
0.502738	36.4828	0.87217786	2.39065575

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LT	1	1325.46786969	1742.45	0.0001
LT*LT	1	1.18028786	1.55	0.2131

SOURCE	DF	TYPE III SS	F VALUE	PR > F
LT	1	14.23054260	18.71	0.0001
LT*LT	1	1.18028786	1.55	0.2131

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	-0.86721488	-2.98	0.0029	0.29099967
LT	0.90318417	4.33	0.0001	0.20881930
LT*LT	0.04459334	1.25	0.2131	0.03579984

Table 11. Regression results on the cubic model LAPE = a + b(LT) + $c(LT)^2 + d(LT)^3$.

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: LAPE LOG (ABSOLUTE PERCENT ERROR+1)

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	1329.88234677	443.29411559	583.85
ERROR	1724	1308.96334775	0.75925948	PR > F
CORRECTED TOTAL	1727	2638.84569452		0.0001

R-SQUARE	C.V.	ROOT MSE	LAPE MEAN
0.503964	36.4484	0.87135497	2.39065575

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LT	1	1325.46786969	1745.74	0.0001
LT*LT	1	1.18028786	1.55	0.2126
LT*LT*LT	1	3.23418922	4.26	0.0392

SOURCE	DF	TYPE III SS	F VALUE	PR > F
LT	1	1.46726572	1.93	0.1647
LT*LT	1	3.49964941	4.61	0.0319
LT*LT*LT	1	3.23418922	4.26	0.0392

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	1.54286725	1.28	0.2000	1.20338098
LT	-1.93063158	-1.39	0.1647	1.38880136
LT*LT	1.08544392	2.15	0.0319	0.50558040
LT*LT*LT	-0.12111800	-2.06	0.0392	0.05868419

Table 12. Regression results on the linear model LAPE = a + b(LN). LAPE is the natural log of (absolute percent error +2), a and b are regression coefficients, and LN is the natural log of the sample interval in minutes.

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: LAPE LOG (ABSOLUTE PERCENT ERROR+2)

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	1	1314.02367916	1314.02367916	1711.93
ERROR	1726	1324.82201536	0.76756780	PR > F
CORRECTED TOTAL	1727	2638.84569452		0.0001

R-SQUARE	C.V.	ROOT MSE	LAPE MEAN
0.497954	36.6472	0.87610947	2.39065575

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LN	1	1314.02367916	1711.93	0.0001

SOURCE	DF	TYPE III SS	F VALUE	PR > F
LN	1	1314.02367916	1711.93	0.0001

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	4.33917747	84.10	0.0001	0.05159460
LN	-1.30856716	-41.38	0.0001	0.03162662

Table 13. Regression results on the quadratic model LAPE = $a + b(LN) + c(LN)^2$.

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: LAPE   LOG (ABSOLUTE PERCENT ERROR+1)

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	2	1316.94990191	658.47495095	859.27
ERROR	1725	1321.89579261	0.76631640	PR > F
CORRECTED TOTAL	1727	2638.84569452		0.0001

R-SQUARE	C.V.	ROOT MSE	LAPE MEAN
0.499063	36.6174	0.87539500	2.39065575

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LN	1	1314.02367916	1714.73	0.0001
LN*LN	1	2.92622275	3.82	0.0508

SOURCE	DF	TYPE III SS	F VALUE	PR > F
LN	1	78.51645282	102.46	0.0001
LN*LN	1	2.92622275	3.82	0.0508

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	4.55485364	37.39	0.0001	0.12181660
LN	-1.61396511	-10.12	0.0001	0.15944756
LN*LN	0.08983264	1.95	0.0508	0.04597106

Table 14. Regression results on the cubic model LAPE = a + b(LN) + $c(LN)^2 + d(LN)^3$.

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: LAPE LOG (ABSOLUTE PERCENT ERROR+1)

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	3	1320.63731126	440.21243709	575.73
ERROR	1724	1318.20838327	0.76462203	PR > F
CORRECTED TOTAL	1727	2638.84569452		0.0001

R-SQUARE	C.V.	ROOT MSE	LAPE MEAN
0.500460	36.5769	0.87442669	2.39065575

SOURCE	DF	TYPE I SS	F VALUE	PR > F
LN	1	1314.02367916	1718.53	0.0001
LN*LN	1	2.92622275	3.83	0.0506
LN*LN*LN	1	3.68740935	4.82	0.0282

SOURCE	DF	TYPE III SS	F VALUE	PR > F
LN	1	0.01061884	0.01	0.9062
LN*LN	1	3.02136495	3.95	0.0470
LN*LN*LN	1	3.68740935	4.82	0.0282

PARAMETER	ESTIMATE	T FOR HO: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	3.87278729	11.61	0.0001	0.33357677
LN	-0.08421019	-0.12	0.9062	0.71457775
LN*LN	-0.90630316	-1.99	0.0470	0.45592699
LN*LN*LN	0.19282472	2.20	0.0282	0.08780625

VITA

I, Jeffrey Lynn Fornear, was born in 1950 in Hammond, IN. I graduated from Enfield High, Enfield, CT in 1968. After enlisting in the Air Force in 1972, I completed a year of specialized electronics classes. For five years I was a technician in Precision Measurement Equipment Laboratories, repairing and calibrating electronic test equipment. Then, for one year, I trained and worked as a civil engineer aide, doing mechanical drafting and construction curveying work. Under Air Force sponsorship I graduated, cum laude, from the University of Utah in 1981 with a B.S. in Meteorology. On a two year assignment as a weather officer at Andersen A.F.B., Guam, I forecasted weather and typhoons across the N. and S. Pacific, and Indian Oceans. I flew typhoon penetration missions and deployed to Okinawa and Australia to support aircraft missions. Since 1983 I have been at Texas A&M University, under full Air Force sponsorship, living in College Station, Texas with my wonderful, supportive wife, Sawang and two amazing daughters, Dawn and Danika.

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